Notation

\( \mathbb{N} \) The set of all natural numbers \( \{1,2,3,\ldots\} \)

\( \mathbb{Z} \) The set of all integers

\( \mathbb{Q} \) The set of all rational numbers

\( \mathbb{R} \) The set of all real numbers

\( S_n \) The group of permutations of \( n \) distinct symbols

\( \mathbb{Z}_n \) \( \{0,1,2,\ldots,n-1\} \) with addition and multiplication modulo \( n \)

\( \emptyset \) empty set

\( A^T \) Transpose of \( A \)

\( i = \sqrt{-1} \)

\( \hat{i}, \hat{j}, \hat{k} \) unit vectors having the directions of the positive \( x, y \) and \( z \) axes of a three dimensional rectangular coordinate system

\( \nabla = \hat{i} \frac{\partial}{\partial x} + \hat{j} \frac{\partial}{\partial y} + \hat{k} \frac{\partial}{\partial z} \)

\( I_n \) Identity matrix of order \( n \)

\( \ln \) logarithm with base \( e \)
SECTION – A
MULTIPLE CHOICE QUESTIONS (MCQ)

Q. 1 – Q.10 carry one mark each.

Q.1 The sequence $\{s_n\}$ of real numbers given by

$$s_n = \frac{\sin \frac{\pi}{2}}{1 \cdot 2} + \frac{\sin \frac{\pi}{2^2}}{2 \cdot 3} + \cdots + \frac{\sin \frac{\pi}{2^n}}{n \cdot (n + 1)}$$

is

(A) a divergent sequence
(B) an oscillatory sequence
(C) not a Cauchy sequence
(D) a Cauchy sequence

Q.2 Let $P$ be the vector space (over $\mathbb{R}$) of all polynomials of degree $\leq 3$ with real coefficients. Consider the linear transformation $T: P \to P$ defined by

$$T(a_0 + a_1x + a_2x^2 + a_3 x^3) = a_3 + a_2x + a_1x^2 + a_0 x^3.$$ 

Then the matrix representation $M$ of $T$ with respect to the ordered basis $\{1, x, x^2, x^3\}$ satisfies

(A) $M^2 + I_4 = 0$
(B) $M^2 - I_4 = 0$
(C) $M - I_4 = 0$
(D) $M + I_4 = 0$

Q.3 Let $f: [-1, 1] \to \mathbb{R}$ be a continuous function. Then the integral

$$\int_0^\pi x f(\sin x) \, dx$$

is equivalent to

(A) $\frac{\pi}{2} \int_0^\pi f(\sin x) \, dx$
(B) $\frac{\pi}{2} \int_0^\pi f(\cos x) \, dx$
(C) $\pi \int_0^\pi f(\sin x) \, dx$
(D) $\pi \int_0^\pi f(\cos x) \, dx$

Q.4 Let $\sigma$ be an element of the permutation group $S_5$. Then the maximum possible order of $\sigma$ is

(A) 5
(B) 6
(C) 10
(D) 15

Q.5 Let $f$ be a strictly monotonic continuous real valued function defined on $[a, b]$ such that $f(a) < a$ and $f(b) > b$. Then which one of the following is TRUE?

(A) There exists exactly one $c \in (a, b)$ such that $f(c) = c$
(B) There exist exactly two points $c_1, c_2 \in (a, b)$ such that $f(c_i) = c_i, \ i = 1, 2$
(C) There exists no $c \in (a, b)$ such that $f(c) = c$
(D) There exist infinitely many points $c \in (a, b)$ such that $f(c) = c$
Q.6 The value of \( \lim_{(x, y) \to (2, -2)} \frac{\sqrt{(x-y)^2}}{x-y-4} \) is

(A) 0  
(B) \( \frac{1}{4} \)  
(C) \( \frac{1}{3} \)  
(D) \( \frac{1}{2} \)

Q.7 Let \( \mathbf{r} = (x \mathbf{i} + y \mathbf{j} + z \mathbf{k}) \) and \( r = |\mathbf{r}| \). If \( f(r) = \ln r \) and \( g(r) = \frac{1}{r}, r \neq 0 \), satisfy

\( 2 \nabla f + h(r) \nabla g = \mathbf{0} \), then \( h(r) \) is

(A) \( r \)  
(B) \( \frac{1}{r} \)  
(C) \( 2r \)  
(D) \( \frac{2}{r} \)

Q.8 The nonzero value of \( n \) for which the differential equation

\[ (3xy^2 + n^2x^2y)dx + (nx^3 + 3x^2y)dy = 0, \quad x \neq 0, \]

becomes exact is

(A) \(-3\)  
(B) \(-2\)  
(C) \(2\)  
(D) \(3\)

Q.9 One of the points which lies on the solution curve of the differential equation

\[ (y - x)dx + (x + y)dy = 0, \]

with the given condition \( y(0) = 1 \), is

(A) \((1, -2)\)  
(B) \((2, -1)\)  
(C) \((2, 1)\)  
(D) \((-1, 2)\)

Q.10 Let \( S \) be a closed subset of \( \mathbb{R} \), \( T \) a compact subset of \( \mathbb{R} \) such that \( S \cap T \neq \emptyset \). Then \( S \cap T \) is

(A) closed but not compact  
(B) not closed  
(C) compact  
(D) neither closed nor compact

Q.11 – Q.30 carry two marks each.

Q.11 Let \( S \) be the series

\[ \sum_{k=1}^{\infty} \frac{1}{(2k-1)2^{(2k-1)}} \]

and \( T \) be the series

\[ \sum_{k=2}^{\infty} \frac{\left(3k-4\right)^{\frac{k+1}{3}}}{3k+2} \]

of real numbers. Then which one of the following is TRUE?

(A) Both the series \( S \) and \( T \) are convergent  
(B) \( S \) is convergent and \( T \) is divergent  
(C) \( S \) is divergent and \( T \) is convergent  
(D) Both the series \( S \) and \( T \) are divergent
Q.12 Let \( \{a_n\} \) be a sequence of positive real numbers satisfying

\[
\frac{4}{a_{n+1}} = \frac{3}{a_n} + \frac{a_n^3}{81}, \quad n \geq 1, \quad a_1 = 1.
\]

Then all the terms of the sequence lie in

(A) \( \left[ \frac{1}{2}, \frac{3}{2} \right] \) \quad (B) \( [0, 1] \) \quad (C) \( [1, 2] \) \quad (D) \( [1, 3] \)

Q.13 The largest eigenvalue of the matrix

\[
\begin{pmatrix}
1 & 4 & 16 \\
4 & 16 & 1 \\
16 & 1 & 4
\end{pmatrix}
\]

is

(A) 16 \quad (B) 21 \quad (C) 48 \quad (D) 64

Q.14 The value of the integral

\[
\frac{(2n)!}{2^{2n} (n!)^2} \int_{-1}^{1} (1 - x^2)^n \, dx, \quad n \in \mathbb{N}
\]

is

(A) \( \frac{2}{(2n+1)!} \) \quad (B) \( \frac{2n}{(2n+1)!} \) \quad (C) \( \frac{2(n!)}{2n+1} \) \quad (D) \( \frac{(n+1)!}{2n+1} \)

Q.15 If the triple integral over the region bounded by the planes

\[
2x + y + z = 4, \quad x = 0, \quad y = 0, \quad z = 0
\]

is given by

\[
\int_{0}^{2} \int_{0}^{\lambda(x)} \int_{0}^{\mu(x,y)} \, dz \, dy \, dx,
\]

then the function \( \lambda(x) - \mu(x, y) \) is

(A) \( x + y \) \quad (B) \( x - y \) \quad (C) \( x \) \quad (D) \( y \)

Q.16 The surface area of the portion of the plane \( y + 2z = 2 \) within the cylinder \( x^2 + y^2 = 3 \) is

(A) \( \frac{3\sqrt{5}}{2} \pi \) \quad (B) \( \frac{5\sqrt{5}}{2} \pi \) \quad (C) \( \frac{7\sqrt{5}}{2} \pi \) \quad (D) \( \frac{9\sqrt{5}}{2} \pi \)
Q.17 Let \( f: \mathbb{R}^2 \rightarrow \mathbb{R} \) be defined by
\[
 f(x, y) = \begin{cases} 
 \frac{xy^2}{x+y} & \text{if } x+y \neq 0 \\
 0 & \text{if } x+y = 0
\end{cases}
\]
Then the value of \( \left( \frac{\partial^2 f}{\partial x \partial y} + \frac{\partial^2 f}{\partial y \partial x} \right) \) at the point \((0, 0)\) is
(A) 0  (B) 1  (C) 2  (D) 4

Q.18 The function \( f(x, y) = 3x^2y + 4y^3 - 3x^2 - 12y^2 + 1 \) has a saddle point at
(A) \((0, 0)\)  (B) \((0, 2)\)  (C) \((1, 1)\)  (D) \((-2, 1)\)

Q.19 Consider the vector field \( \vec{F} = r^\beta (y \hat{i} - x \hat{j}) \), where \( \beta \in \mathbb{R} \), \( \vec{r} = xi + yj \) and \( r = |\vec{r}| \). If the absolute value of the line integral \( \oint_C \vec{F} \cdot d\vec{r} \) along the closed curve \( C: x^2 + y^2 = a^2 \) (oriented counter clockwise) is \( 2\pi \), then \( \beta \) is
(A) \(-2\)  (B) \(-1\)  (C) \(1\)  (D) \(2\)

Q.20 Let \( S \) be the surface of the cone \( z = \sqrt{x^2 + y^2} \) bounded by the planes \( z = 0 \) and \( z = 3 \). Further, let \( C \) be the closed curve forming the boundary of the surface \( S \). A vector field \( \vec{F} \) is such that \( \nabla \times \vec{F} = -xi - yj \). The absolute value of the line integral \( \oint_C \vec{F} \cdot d\vec{r} \), where \( \vec{r} = xi + yj + zk \) and \( r = |\vec{r}| \), is
(A) 0  (B) \(9\pi\)  (C) \(15\pi\)  (D) \(18\pi\)

Q.21 Let \( y(x) \) be the solution of the differential equation
\[
\frac{d}{dx} \left( x \frac{dy}{dx} \right) = x; \quad y(1) = 0, \quad \left. \frac{dy}{dx} \right|_{x=1} = 0
\]
Then \( y(2) \) is
(A) \(\frac{3}{4} + \frac{1}{2} \ln 2\)  (B) \(\frac{3}{4} - \frac{1}{2} \ln 2\)
(C) \(\frac{3}{4} + \ln 2\)  (D) \(\frac{3}{4} - \ln 2\)

Q.22 The general solution of the differential equation with constant coefficients
\[
\frac{d^2 y}{dx^2} + b \frac{dy}{dx} + cy = 0
\]
approaches zero as \( x \to \infty \), if
(A) \( b \) is negative and \( c \) is positive
(B) \( b \) is positive and \( c \) is negative
(C) both \( b \) and \( c \) are positive
(D) both \( b \) and \( c \) are negative
Q.23 Let $S \subseteq \mathbb{R}$ and $\partial S$ denote the set of points $x$ in $\mathbb{R}$ such that every neighbourhood of $x$ contains some points of $S$ as well as some points of complement of $S$. Further, let $\bar{S}$ denote the closure of $S$. Then which one of the following is FALSE?

(A) $\partial \mathbb{Q} = \mathbb{R}$  
(B) $\partial (\mathbb{R} \setminus T) = \partial T$, $T \subset \mathbb{R}$  
(C) $\partial (T \cup V) = \partial T \cup \partial V$, $T, V \subset \mathbb{R}$, $T \cap V \neq \emptyset$  
(D) $\partial T = T \cap (\mathbb{R} \setminus \bar{T})$, $T \subset \mathbb{R}$

Q.24 The sum of the series

$$\sum_{n=2}^{\infty} \frac{(-1)^n}{n^2 + n - 2}$$

is

(A) $\frac{1}{3} \ln 2 - \frac{5}{18}$  
(B) $\frac{1}{3} \ln 2 - \frac{5}{6}$  
(C) $\frac{2}{3} \ln 2 - \frac{5}{18}$  
(D) $\frac{2}{3} \ln 2 - \frac{5}{6}$

Q.25 Let $f(x) = \frac{1}{1+|x|} + \frac{1}{1+|x-1|}$ for all $x \in [-1, 1]$. Then which one of the following is TRUE?

(A) Maximum value of $f(x)$ is $\frac{3}{2}$  
(B) Minimum value of $f(x)$ is $\frac{1}{3}$  
(C) Maximum of $f(x)$ occurs at $x = \frac{1}{2}$  
(D) Minimum of $f(x)$ occurs at $x = 1$

Q.26 The matrix $M = \begin{bmatrix} \cos \alpha & \sin \alpha \\ i \sin \alpha & i \cos \alpha \end{bmatrix}$ is a unitary matrix when $\alpha$ is

(A) $(2n + 1)\frac{\pi}{2}$, $n \in \mathbb{Z}$  
(B) $(3n + 1)\frac{\pi}{3}$, $n \in \mathbb{Z}$  
(C) $(4n + 1)\frac{\pi}{4}$, $n \in \mathbb{Z}$  
(D) $(5n + 1)\frac{\pi}{5}$, $n \in \mathbb{Z}$

Q.27 Let $M = \begin{bmatrix} 0 & 1 & -2 \\ -1 & 0 & \alpha \\ 2 & -\alpha & 0 \end{bmatrix}$, $\alpha \in \mathbb{R} \setminus \{0\}$ and $b$ a non-zero vector such that $Mx = b$ for some $x \in \mathbb{R}^3$. Then the value of $x^Tb$ is

(A) $-\alpha$  
(B) $\alpha$  
(C) $0$  
(D) $1$

Q.28 The number of group homomorphisms from the cyclic group $\mathbb{Z}_4$ to the cyclic group $\mathbb{Z}_7$ is

(A) 7  
(B) 3  
(C) 2  
(D) 1

Q.29 In the permutation group $S_n$ ($n \geq 5$), if $H$ is the smallest subgroup containing all the 3-cycles, then which one of the following is TRUE?

(A) Order of $H$ is 2  
(B) Index of $H$ in $S_n$ is 2  
(C) $H$ is abelian  
(D) $H = S_n$
Q.30 Let \( f: \mathbb{R} \to \mathbb{R} \) be defined as
\[
f(x) = \begin{cases} 
  x(1 + x^\alpha \sin(\ln x^2)) & \text{if } x \neq 0 \\
  0 & \text{if } x = 0.
\end{cases}
\]

Then, at \( x = 0 \), the function \( f \) is

(A) continuous and differentiable when \( \alpha = 0 \)
(B) continuous and differentiable when \( \alpha > 0 \)
(C) continuous and differentiable when \(-1 < \alpha < 0\)
(D) continuous and differentiable when \( \alpha < -1 \)

SECTION - B
MULTIPLE SELECT QUESTIONS (MSQ)

Q.31 – Q.40 carry two marks each.

Q.31 Let \( \{s_n\} \) be a sequence of positive real numbers satisfying
\[
2 s_{n+1} = s_n^2 + \frac{3}{4}, \quad n \geq 1.
\]

If \( \alpha \) and \( \beta \) are the roots of the equation \( x^2 - 2x + \frac{3}{4} = 0 \) and \( \alpha < s_1 < \beta \), then which of the following statement(s) is(are) TRUE?

(A) \( \{s_n\} \) is monotonically decreasing
(B) \( \{s_n\} \) is monotonically increasing
(C) \( \lim_{n \to \infty} s_n = \alpha \)
(D) \( \lim_{n \to \infty} s_n = \beta \)

Q.32 The value(s) of the integral
\[
\int_{-\pi}^{\pi} |x| \cos nx \, dx, \quad n \geq 1
\]
is (are)

(A) 0 when \( n \) is even
(B) 0 when \( n \) is odd
(C) \(-\frac{4}{n^2}\) when \( n \) is even
(D) \(-\frac{4}{n^2}\) when \( n \) is odd
Q.33 Let $f: \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x, y) = \begin{cases} \frac{xy}{|x|} & \text{if } x \neq 0 \\ 0 & \text{elsewhere} \end{cases}.$$ 

Then at the point $(0, 0)$, which of the following statement(s) is(are) TRUE?

(A) $f$ is not continuous  
(B) $f$ is continuous  
(C) $f$ is differentiable  
(D) Both first order partial derivatives of $f$ exist

Q.34 Consider the vector field $\mathbf{F} = x\mathbf{i} + y\mathbf{j}$ on an open connected set $S \subset \mathbb{R}^2$. Then which of the following statement(s) is(are) TRUE?

(A) Divergence of $\mathbf{F}$ is zero on $S$  
(B) The line integral of $\mathbf{F}$ is independent of path in $S$  
(C) $\mathbf{F}$ can be expressed as a gradient of a scalar function on $S$  
(D) The line integral of $\mathbf{F}$ is zero around any piecewise smooth closed path in $S$

Q.35 Consider the differential equation

$$\sin 2x \frac{dy}{dx} = 2y + 2 \cos x, \quad y\left(\frac{\pi}{4}\right) = 1 - \sqrt{2}.$$

Then which of the following statement(s) is(are) TRUE?

(A) The solution is unbounded when $x \to 0$  
(B) The solution is unbounded when $x \to \frac{\pi}{2}$  
(C) The solution is bounded when $x \to 0$  
(D) The solution is bounded when $x \to \frac{\pi}{2}$

Q.36 Which of the following statement(s) is(are) TRUE?

(A) There exists a connected set in $\mathbb{R}$ which is not compact  
(B) Arbitrary union of closed intervals in $\mathbb{R}$ need not be compact  
(C) Arbitrary union of closed intervals in $\mathbb{R}$ is always closed  
(D) Every bounded infinite subset $V$ of $\mathbb{R}$ has a limit point in $V$ itself

Q.37 Let $P(x) = \left(\frac{5}{13}\right)^x + \left(\frac{12}{13}\right)^x - 1$ for all $x \in \mathbb{R}$. Then which of the following statement(s) is(are) TRUE?

(A) The equation $P(x) = 0$ has exactly one solution in $\mathbb{R}$  
(B) $P(x)$ is strictly increasing for all $x \in \mathbb{R}$  
(C) The equation $P(x) = 0$ has exactly two solutions in $\mathbb{R}$  
(D) $P(x)$ is strictly decreasing for all $x \in \mathbb{R}$
Q.38 Let $G$ be a finite group and $o(G)$ denotes its order. Then which of the following statement(s) is(are) TRUE?

(A) $G$ is abelian if $o(G) = pq$ where $p$ and $q$ are distinct primes
(B) $G$ is abelian if every non identity element of $G$ is of order 2
(C) $G$ is abelian if the quotient group $\frac{G}{Z(G)}$ is cyclic, where $Z(G)$ is the center of $G$
(D) $G$ is abelian if $o(G) = p^3$, where $p$ is prime

Q.39 Consider the set $V = \left\{ \begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3 \mid ax + \beta y + z = \gamma, \, \alpha, \beta, \gamma \in \mathbb{R} \right\}$. For which of the following choice(s) the set $V$ becomes a two dimensional subspace of $\mathbb{R}^3$ over $\mathbb{R}$?

(A) $\alpha = 0, \beta = 1, \gamma = 0$
(B) $\alpha = 0, \beta = 1, \gamma = 1$
(C) $\alpha = 1, \beta = 0, \gamma = 0$
(D) $\alpha = 1, \beta = 1, \gamma = 0$

Q.40 Let $S = \left\{ \frac{1}{(3^n)} + \frac{1}{7^n} \mid n, m \in \mathbb{N} \right\}$. Then which of the following statement(s) is(are) TRUE?

(A) $S$ is closed
(B) $S$ is not open
(C) $S$ is connected
(D) $0$ is a limit point of $S$

SECTION – C
NUMERICAL ANSWER TYPE (NAT)

Q. 41 – Q. 50 carry one mark each.

Q.41 Let $\{s_n\}$ be a sequence of real numbers given by

$$s_n = 2^{(-1)^n} \left(1 - \frac{1}{n}\right) \sin \frac{n\pi}{2}, \quad n \in \mathbb{N}.$$ 

Then the least upper bound of the sequence $\{s_n\}$ is ____________

Q.42 Let $\{s_k\}$ be a sequence of real numbers, where

$$s_k = k^{\alpha/k}, \quad k \geq 1, \quad \alpha > 0.$$ 

Then

$$\lim_{n \to \infty} \left( s_1 \cdot s_2 \cdots s_n \right)^{1/n}$$

is ____________
Q.43 Let \( \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3 \) be a non-zero vector and \( A = \frac{\mathbf{x} \cdot \mathbf{x}}{\mathbf{x} \cdot \mathbf{x}} \). Then the dimension of the vector space \( \{ \mathbf{y} \in \mathbb{R}^3 \mid A\mathbf{y} = \mathbf{0} \} \) over \( \mathbb{R} \) is ____________

Q.44 Let \( f \) be a real valued function defined by

\[
f(x, y) = 2 \ln \left( x^2 y^2 e^x \right), \quad x > 0, y > 0.
\]

Then the value of \( x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} \) at any point \((x, y)\), where \( x > 0, y > 0, \) is ____________

Q.45 Let \( \mathbf{F} = \sqrt{x} \hat{i} + (x + y^3) \hat{j} \) be a vector field for all \((x, y)\) with \( x \geq 0 \) and \( \mathbf{r} = xf \hat{i} + yf \hat{j} \). Then the value of the line integral \( \int_C \mathbf{F} \cdot d\mathbf{r} \) from \((0, 0)\) to \((1, 1)\) along the path \( C: x = t^2, y = t^3, 0 \leq t \leq 1 \) is ____________

Q.46 If \( f: (-1, \infty) \to \mathbb{R} \) defined by \( f(x) = \frac{x}{1+x} \) is expressed as

\[
f(x) = \frac{2}{3} + \frac{1}{9} (x - 2) + \frac{c(x - 2)^2}{(1 + \xi)^3},
\]

where \( \xi \) lies between 2 and \( x \), then the value of \( c \) is ____________

Q.47 Let \( y_1(x), y_2(x) \) and \( y_3(x) \) be linearly independent solutions of the differential equation

\[
\frac{d^3y}{dx^3} - 6 \frac{d^2y}{dx^2} + 11 \frac{dy}{dx} - 6y = 0.
\]

If the Wronskian \( W(y_1, y_2, y_3) \) is of the form \( ke^{bx} \) for some constant \( k \), then the value of \( b \) is ____________

Q.48 The radius of convergence of the power series

\[
\sum_{n=1}^{\infty} \frac{(-4)^n}{n(n + 1)} (x + 2)^{2n}
\]

is ____________
Q.49 Let \( f: (0, \infty) \to \mathbb{R} \) be a continuous function such that
\[
\int_0^x f(t) \, dt = -2 + \frac{x^2}{2} + 4x \sin 2x + 2 \cos 2x.
\]
Then the value of \( \frac{1}{\pi} f\left(\frac{\pi}{4}\right) \) is __________.

Q.50 Let \( G \) be a cyclic group of order 12. Then the number of non-isomorphic subgroups of \( G \) is __________

Q.51 – Q.60 carry two marks each.

Q.51 The value of \( \lim_{n \to \infty} \left( 8n - \frac{1}{n} \right)^{\frac{(-1)^n}{n^2}} \) is equal to ________

Q.52 Let \( R \) be the region enclosed by \( x^2 + 4y^2 \geq 1 \) and \( x^2 + y^2 \leq 1 \). Then the value of
\[
\iint_R |xy| \, dx \, dy
\]
is ________

Q.53 Let
\[
M = \begin{bmatrix} \alpha & 1 & 1 \\ 1 & \beta & 1 \\ 1 & 1 & \gamma \end{bmatrix}, \quad \alpha \beta \gamma = 1, \quad \alpha, \beta, \gamma \in \mathbb{R} \quad \text{and} \quad x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \in \mathbb{R}^3.
\]
Then \( Mx = 0 \) has infinitely many solutions if \( \text{trace}(M) \) is ________

Q.54 Let \( C \) be the boundary of the region enclosed by \( y = x^2, \ y = x + 2, \) and \( x = 0 \). Then the value of the line integral
\[
\oint_C (xy - y^2)dx - x^3dy,
\]
where \( C \) is traversed in the counter clockwise direction, is ________
Q.55  Let S be the closed surface forming the boundary of the region V bounded by \( x^2 + y^2 = 3 \), \( z = 0 \), \( z = 6 \). A vector field \( \mathbf{F} \) is defined over V with \( \nabla \cdot \mathbf{F} = 2y + z + 1 \). Then the value of

\[
\frac{1}{\pi} \int_{S} \mathbf{F} \cdot \hat{n} \, ds,
\]

where \( \hat{n} \) is the unit outward drawn normal to the surface \( S \), is ___________.

Q.56  Let \( y(x) \) be the solution of the differential equation

\[
\frac{d^2 y}{dx^2} + 5 \frac{dy}{dx} + 6y = 0, \quad y(0) = 1, \quad \frac{dy}{dx} \Big|_{x=0} = -1.
\]

Then \( y(x) \) attains its maximum value at \( x = \) ____________

Q.57  The value of the double integral

\[
\int_{0}^{\pi} \int_{0}^{x} \sin y \, dy \, dx
\]

is ____________

Q.58  Let \( H \) denote the group of all \( 2 \times 2 \) invertible matrices over \( \mathbb{Z}_5 \) under usual matrix multiplication. Then the order of the matrix \( \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} \) in \( H \) is ____________

Q.59  Let \( A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 5 & 2 \end{bmatrix} \); \( B = \begin{bmatrix} 1 & 2 \\ -1 & 0 \\ 3 & 1 \end{bmatrix} \). \( N(A) \) the null space of \( A \) and \( R(B) \) the range space of \( B \). Then the dimension of \( N(A) \cap R(B) \) over \( \mathbb{R} \) is ____________

Q.60  The maximum value of \( f(x,y) = x^2 + 2y^2 \) subject to the constraint \( y - x^2 + 1 = 0 \) is ____________
<table>
<thead>
<tr>
<th>Qn. No.</th>
<th>Qn. Type</th>
<th>Key(s)</th>
<th>Mark(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>MCQ</td>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>MCQ</td>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>3</td>
<td>MCQ</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>4</td>
<td>MCQ</td>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>5</td>
<td>MCQ</td>
<td>A</td>
<td>1</td>
</tr>
<tr>
<td>6</td>
<td>MCQ</td>
<td>B</td>
<td>1</td>
</tr>
<tr>
<td>7</td>
<td>MCQ</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>MCQ</td>
<td>D</td>
<td>1</td>
</tr>
<tr>
<td>9</td>
<td>MCQ</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>10</td>
<td>MCQ</td>
<td>C</td>
<td>1</td>
</tr>
<tr>
<td>11</td>
<td>MCQ</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>12</td>
<td>MCQ</td>
<td>D</td>
<td>2</td>
</tr>
<tr>
<td>13</td>
<td>MCQ</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>14</td>
<td>MCQ</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>15</td>
<td>MCQ</td>
<td>D</td>
<td>2</td>
</tr>
<tr>
<td>16</td>
<td>MCQ</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>17</td>
<td>MCQ</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>18</td>
<td>MCQ</td>
<td>D</td>
<td>2</td>
</tr>
<tr>
<td>19</td>
<td>MCQ</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>20</td>
<td>MCQ</td>
<td>MTA</td>
<td>2</td>
</tr>
<tr>
<td>21</td>
<td>MCQ</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>22</td>
<td>MCQ</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>23</td>
<td>MCQ</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>24</td>
<td>MCQ</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>25</td>
<td>MCQ</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>26</td>
<td>MCQ</td>
<td>A</td>
<td>2</td>
</tr>
<tr>
<td>27</td>
<td>MCQ</td>
<td>C</td>
<td>2</td>
</tr>
<tr>
<td>28</td>
<td>MCQ</td>
<td>D</td>
<td>2</td>
</tr>
<tr>
<td>29</td>
<td>MCQ</td>
<td>B</td>
<td>2</td>
</tr>
<tr>
<td>30</td>
<td>MCQ</td>
<td>MTA</td>
<td>2</td>
</tr>
<tr>
<td>31</td>
<td>MSQ</td>
<td>A;C</td>
<td>2</td>
</tr>
<tr>
<td>32</td>
<td>MSQ</td>
<td>A;D</td>
<td>2</td>
</tr>
<tr>
<td>33</td>
<td>MSQ</td>
<td>B;D</td>
<td>2</td>
</tr>
<tr>
<td>34</td>
<td>MSQ</td>
<td>B;C;D</td>
<td>2</td>
</tr>
<tr>
<td>35</td>
<td>MSQ</td>
<td>C;D</td>
<td>2</td>
</tr>
<tr>
<td>36</td>
<td>MSQ</td>
<td>A;B</td>
<td>2</td>
</tr>
<tr>
<td>37</td>
<td>MSQ</td>
<td>A;D</td>
<td>2</td>
</tr>
<tr>
<td>38</td>
<td>MSQ</td>
<td>B;C</td>
<td>2</td>
</tr>
<tr>
<td>39</td>
<td>MSQ</td>
<td>A;C;D</td>
<td>2</td>
</tr>
<tr>
<td>40</td>
<td>MSQ</td>
<td>B;D</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Qn. No.</th>
<th>Qn. Type</th>
<th>Key(s)</th>
<th>Mark(s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>41</td>
<td>NAT</td>
<td>0.5:0.5</td>
<td>1</td>
</tr>
<tr>
<td>42</td>
<td>NAT</td>
<td>1.0:1.0</td>
<td>1</td>
</tr>
<tr>
<td>43</td>
<td>NAT</td>
<td>2.0:2.0</td>
<td>1</td>
</tr>
<tr>
<td>44</td>
<td>NAT</td>
<td>8.0:8.0</td>
<td>1</td>
</tr>
<tr>
<td>45</td>
<td>NAT</td>
<td>1.49:1.55</td>
<td>1</td>
</tr>
<tr>
<td>46</td>
<td>NAT</td>
<td>-1:-1</td>
<td>1</td>
</tr>
<tr>
<td>47</td>
<td>NAT</td>
<td>6.0:6.0</td>
<td>1</td>
</tr>
<tr>
<td>48</td>
<td>NAT</td>
<td>0.5:0.5</td>
<td>1</td>
</tr>
<tr>
<td>49</td>
<td>NAT</td>
<td>0.25:0.25</td>
<td>1</td>
</tr>
<tr>
<td>50</td>
<td>NAT</td>
<td>6.0:6.0</td>
<td>1</td>
</tr>
<tr>
<td>51</td>
<td>NAT</td>
<td>1.0:1.0</td>
<td>2</td>
</tr>
<tr>
<td>52</td>
<td>NAT</td>
<td>0.35:0.4</td>
<td>2</td>
</tr>
<tr>
<td>53</td>
<td>NAT</td>
<td>3.0:3.0</td>
<td>2</td>
</tr>
<tr>
<td>54</td>
<td>NAT</td>
<td>0.8:1.9</td>
<td>2</td>
</tr>
<tr>
<td>55</td>
<td>NAT</td>
<td>72.0:72.0</td>
<td>2</td>
</tr>
<tr>
<td>56</td>
<td>NAT</td>
<td>-0.3:-0.25</td>
<td>2</td>
</tr>
<tr>
<td>57</td>
<td>NAT</td>
<td>2.0:2.0</td>
<td>2</td>
</tr>
<tr>
<td>58</td>
<td>NAT</td>
<td>3.0:3.0</td>
<td>2</td>
</tr>
<tr>
<td>59</td>
<td>NAT</td>
<td>1.0:1.0</td>
<td>2</td>
</tr>
<tr>
<td>60</td>
<td>NAT</td>
<td>2.0:2.0</td>
<td>2</td>
</tr>
</tbody>
</table>